The Evolution of Erasure Codes for Large Scale Data Storage and Multimedia Broadcast

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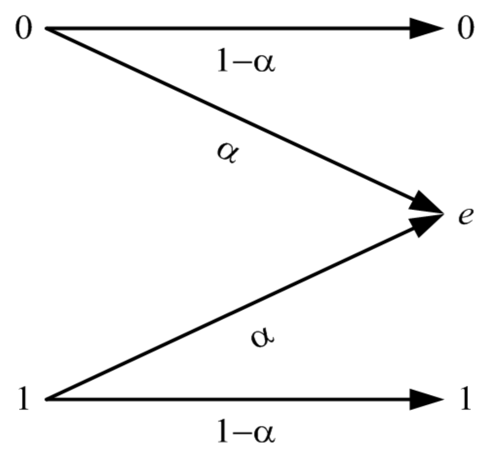
Contents

- Fundamentals
- Efficient Erasure Codes
  - Rateless Codes, Raptor codes, Online codes
- Use of erasure codes for Disk array storage
- Reliability of large scale storage using optimal/sub-optimal erasure codes
- Optimization of rateless codes for progressive source transmission
- Conclusions

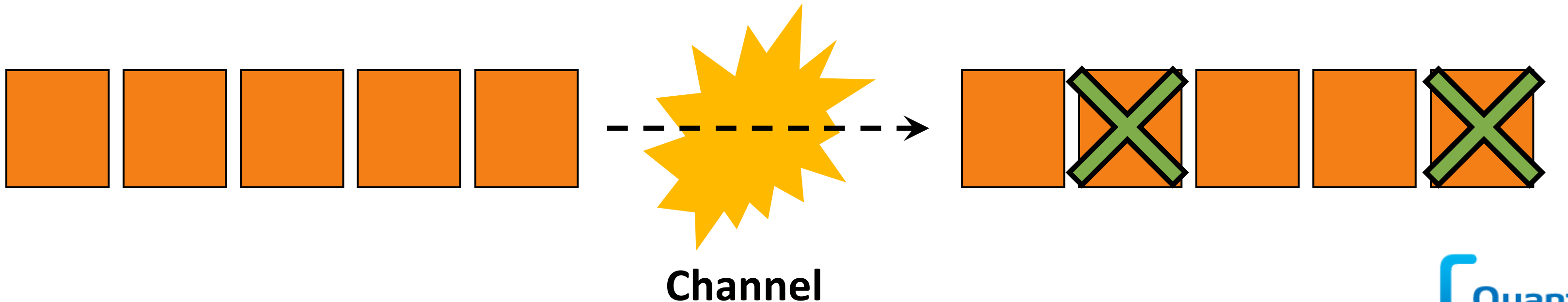
Channel Model

- Binary Erasure Channel



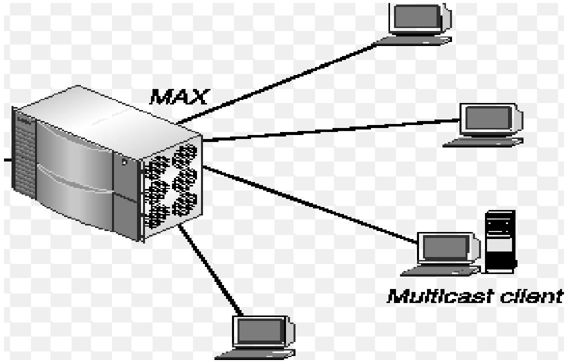
- *e : Erasure*
- *α : erasure probability*
- *Capacity: 1- α*
- *An average of α fraction of bits are lost in the channel, we can at most recover a proportion (1- α) of the bits.*

- Message bits are typically packetized.
- Packets may be corrupted or lost during transmission.



Possibilities for lost data recovery

- If there is a feedback channel, request the lost packets to be retransmitted.
- This may result in round-trip delays and lots of feedback channel use.
- For broadcast, number of request might be overwhelming!



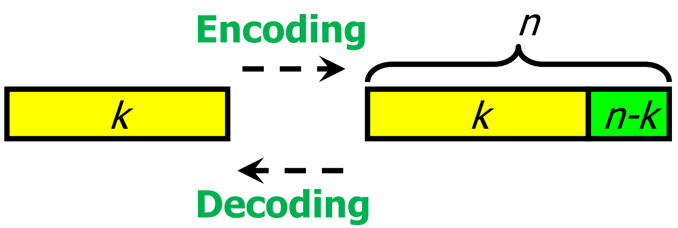
- Forward Error Correction (Erasure coding to restore data).

Coding theory basics

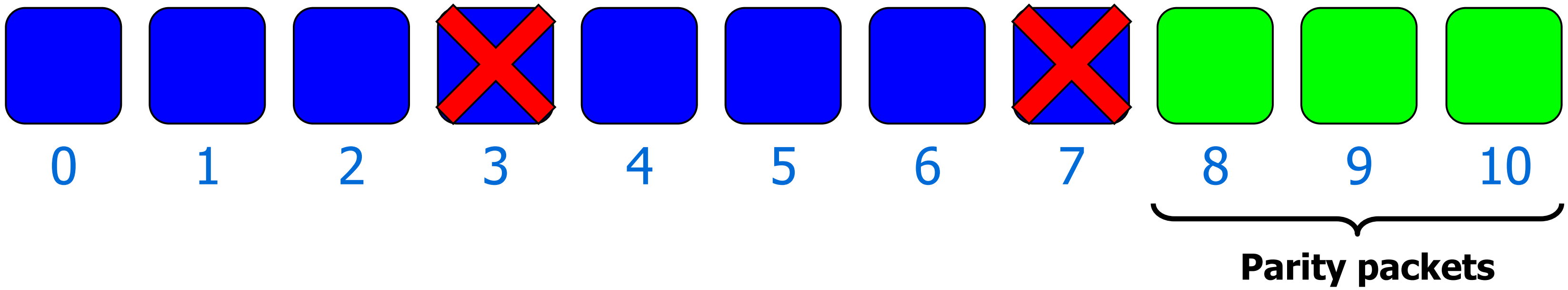
- A code C over a finite alphabet $\sum$ of length n is a subset of $\sum^n$
  - The elements of C are called the codewords in C.
  - If $|\sum| = q$, C is called q-ary code.
- A binary code $(q = 2)$ is a code over the alphabet $\{0, 1\}$.
  - $C_1 = \{000, 010, 101, 100\}$
  - $C_2 = \{00000, 01101, 10111, 11011\}$
- The maping between codewords and message sequences is called “encoding”. The reverse of this operation is called “decoding”.
- Hamming distance:
  - $h(x, y) =$ the number of symbols *x* and *y* differ. $h(10101, 01100) = 3$,
- Minimal distance of a code C:
  - $d_{min}(C) = \min\{h(x, y) | x, y \in C, x \neq y\}$,
- **Theorem 1:** A code with minimal distance d_{min} can correct $d_{min} - 1$ erasures.

Classical Erasure codes

- Encode an original data of k packets to n code packets. Such a code is referred as (n, k) block code.
- **Theorem 2:** A (n, k) code with minimal distance d_{min} satisfies $d_{min} \leq n - k + 1$. (Singleton bound)
- The decoder needs $\tilde{n} \geq k$ code packets to reconstruct the original data.
- If $\tilde{n} = k$, the code is called maximum distance separable (MDS)
- Redundant packets: $n - k$.
- Rate of the code: $r = k/n$
- Overhead: $c = n/k$



Example: MDS block code (11,8)



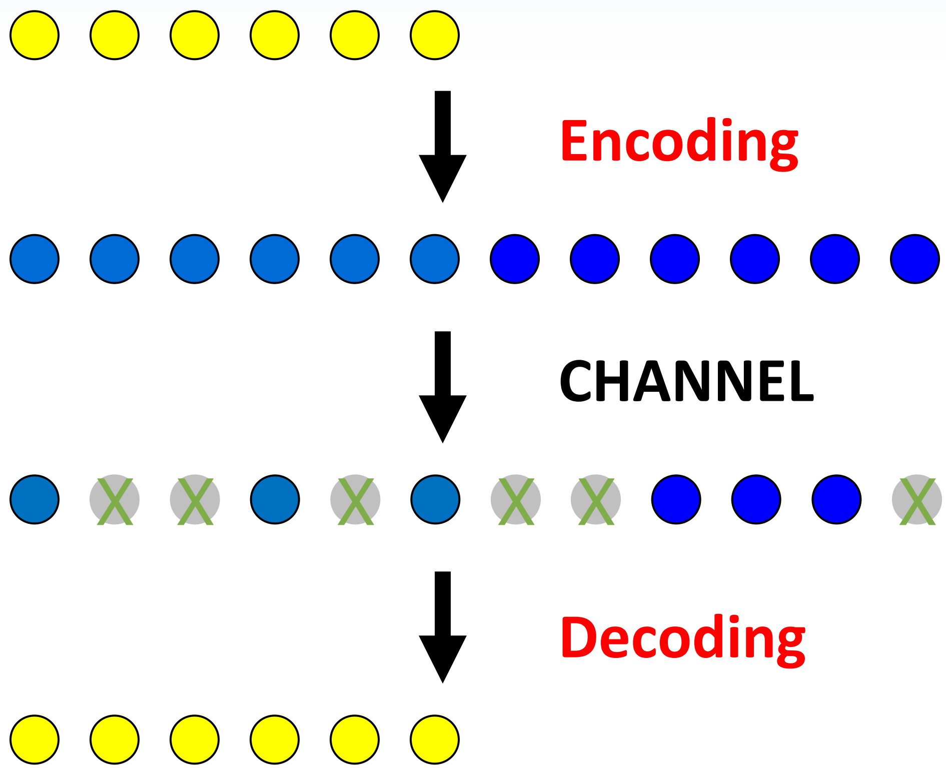
Classical Erasure codes

k message

$n = ck$ encoding

$\geq k$ collected

k message



Trivial Binary MDS Erasure codes

- Repetition coding: *(n, k=1)*

I	I	I	I	I	I	I	I

- Parity coding: *(n, k=n-1)*

I_1	I_2	I_3	I_4	I_5	I_6	I_7	P

$$P=\sum_{i=1}^{7} I_i \pmod 2$$

- There is no non-trivial binary MDS code.

Non-Trivial MDS Erasure codes

- One of the well known **non-trivial, non-binary** MDS code is Reed-Solomon (RS) codes. RS codes are defined over Galois Fields such as $GF(2^m)$.
- Construction is based on a polynomial evaluation.

$$\mathbf{m} = (m_0, m_1, \ldots, m_{k-1})^T$$

$$m(x) = m_0 + m_1 x + m_2 + \cdots + m_{k-1} x^{k-1}$$

- Evaluate $m(x)$ at n specific points to form the codeword:

$$\mathbf{c} = (c_0, c_1, \ldots, c_{n-1})$$

- Encoding Complexity ~ $O(ck^2)$
- Decoding Complexity ~ $O(n \log^2(n) \log(\log(n)))$

Not very easy

- Complexity is also a strong function of the size of the Galois Field over which the code is defined. If RS code is defined over $GF(2^m)$ and $n = 2^m - 1$, decoding complexity can be approximated by

$$C = 2m^2(N_{multiplications} + N_{inversions}) + mN_{additions} \qquad [1]$$

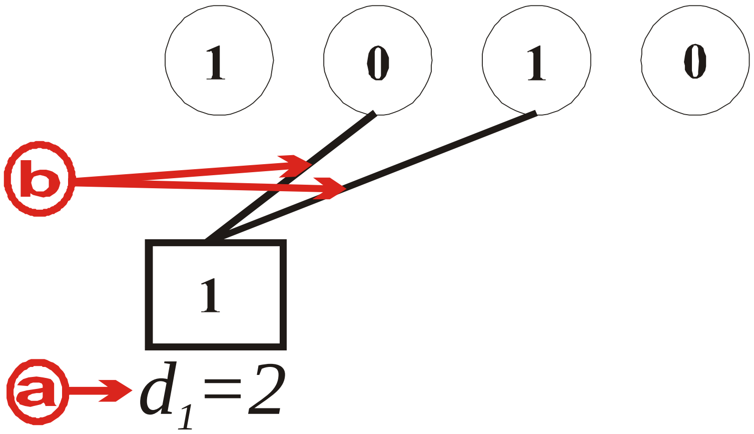
[1] N. Chen and Z. Yan, "Complexity analysis of reed-solomon decoding over GF(2m) without using syndromes," *EURASIP Journal on Wireless Communications and Networking*, vol. 2008, Article ID 843634, 11 pages, 2008.

Issues with conventional erasure codes

- RS code are <u>fixed rate</u> code and thus, its rate must be fixed before transmission.
- In a broadcast scenario, the erasure rate of the channel is not known prior to transmission.
- RS codes are complex to implement, particularly for large block lengths n and rate r.
- If an erased symbol is to be reconstructed, all data must be read. In a storage scenario, if each data packet is stored in different nodes, all nodes must be accessed → increased bandwidth usage.
- We need a different paradigm for constructing erasure codes, possibly with MDS property!

Rateless erasure codes

- Rate-less codes, i.e. there is no predetermined overhead $c=n/k$. One can generate as much code symbols as desired. An instantiation of such a construction is given by Luby in 2001, called Luby Transform (LT) codes.
- Asymptotically optimal: Only $n = (1+\epsilon)k$ coded symbols are enough to recover all k information symbols.
- **Simple Encoding:** encoded symbols are XORs of data symbols.
  - Pick a degree d from the appropriate degree distribution $\Omega(x)$ **(a)**
  - Randomly pick d data symbols. **(b)**
  - Encode them as encoded symbol by using XOR operations.



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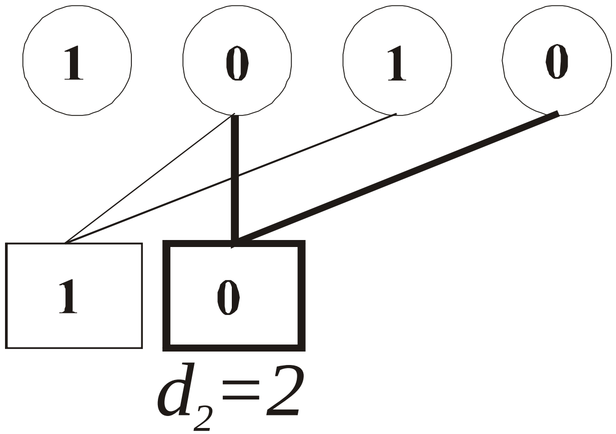

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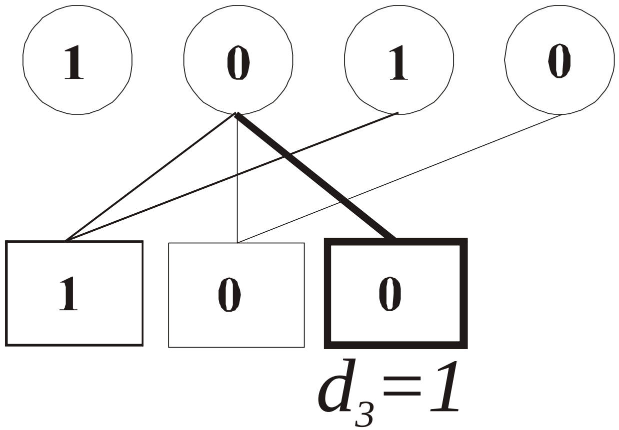

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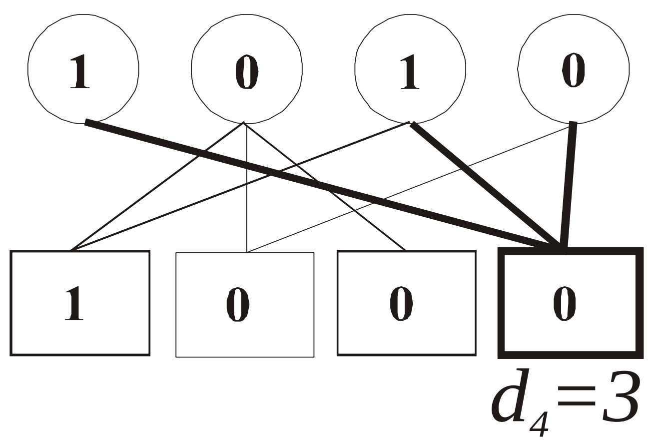

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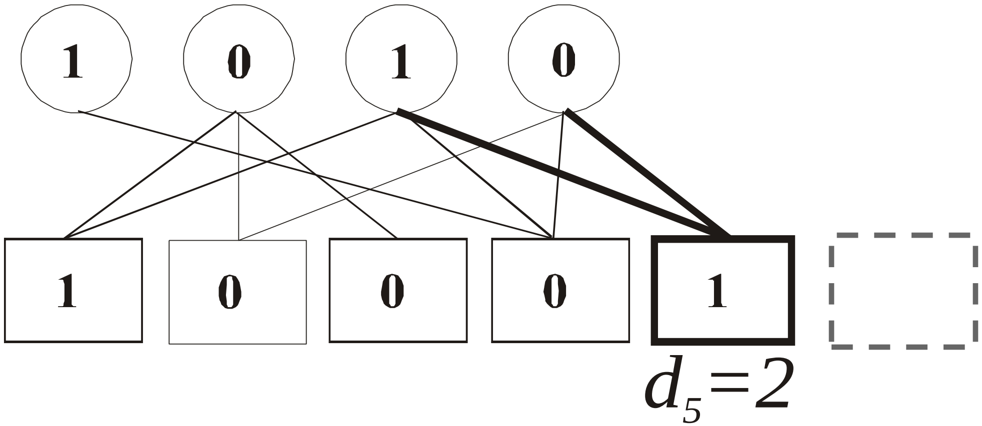
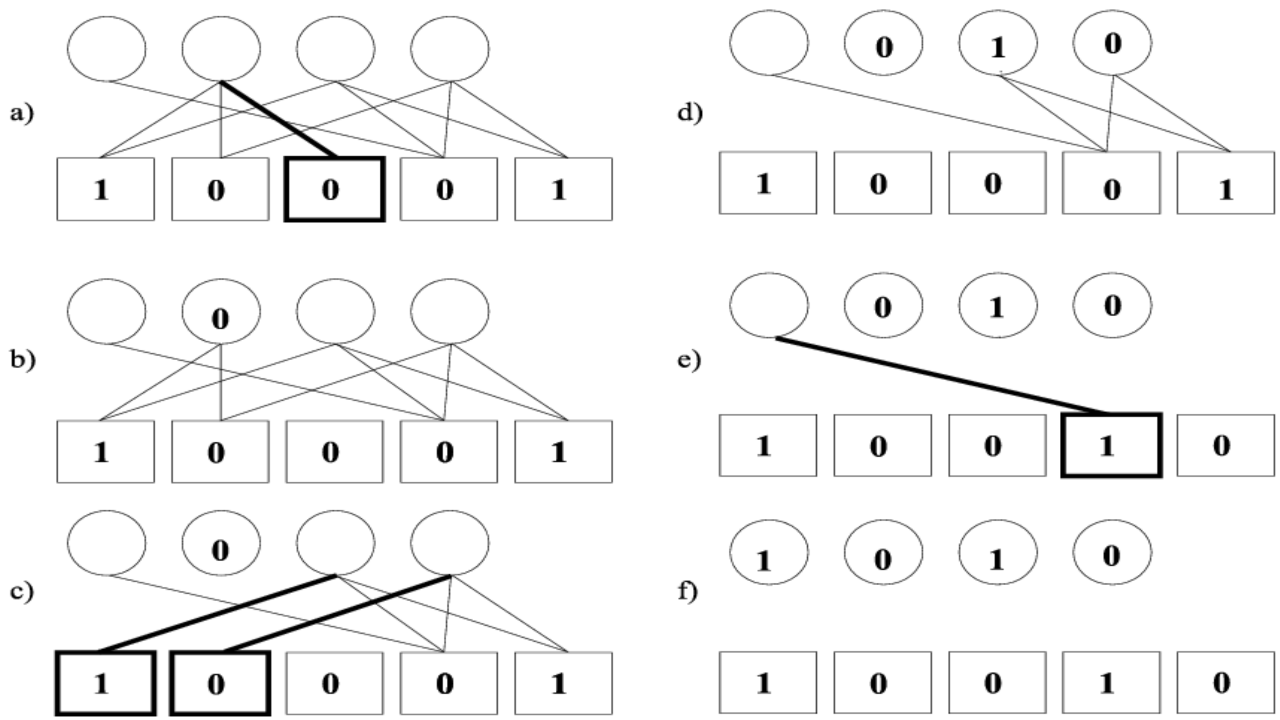


Simple Decoding

- Coded symbols are sent over a binary erasure channel.
- Decoder uses a Belief Propagation (BP) algorithm.



Degree distribution

Soliton Distribution

$$\underbrace{\qquad\qquad\qquad}$$

$$\rho(i) = \begin{cases} 1/k & i = 1 \\ 1/i(i-1) & i \in \{1..k\} \end{cases}$$

Robust Soliton Distribution (RSD)

$$\underbrace{\qquad\qquad\qquad}$$

$$R = c \cdot \ln(k/\delta)\sqrt{k}, \qquad c, \delta > 0$$

$$\tau(i) = \begin{cases} R/ik & i \in \{1 \dots k/R - 1\} \\ \dfrac{R \cdot \ln(R/\delta)}{k} & i = k/R \\ 0 & i \in \{k/R + 1..k\} \end{cases}$$

$$\beta = \sum_{i=1}^{k} \rho(i) + \tau(i)$$

$$\Rightarrow \mu(i) = \frac{\rho(i) + \tau(i)}{\beta}$$

- Degree distribution is chosen such that the decoding of the whole message block (k symbols) is ensured with high probability.

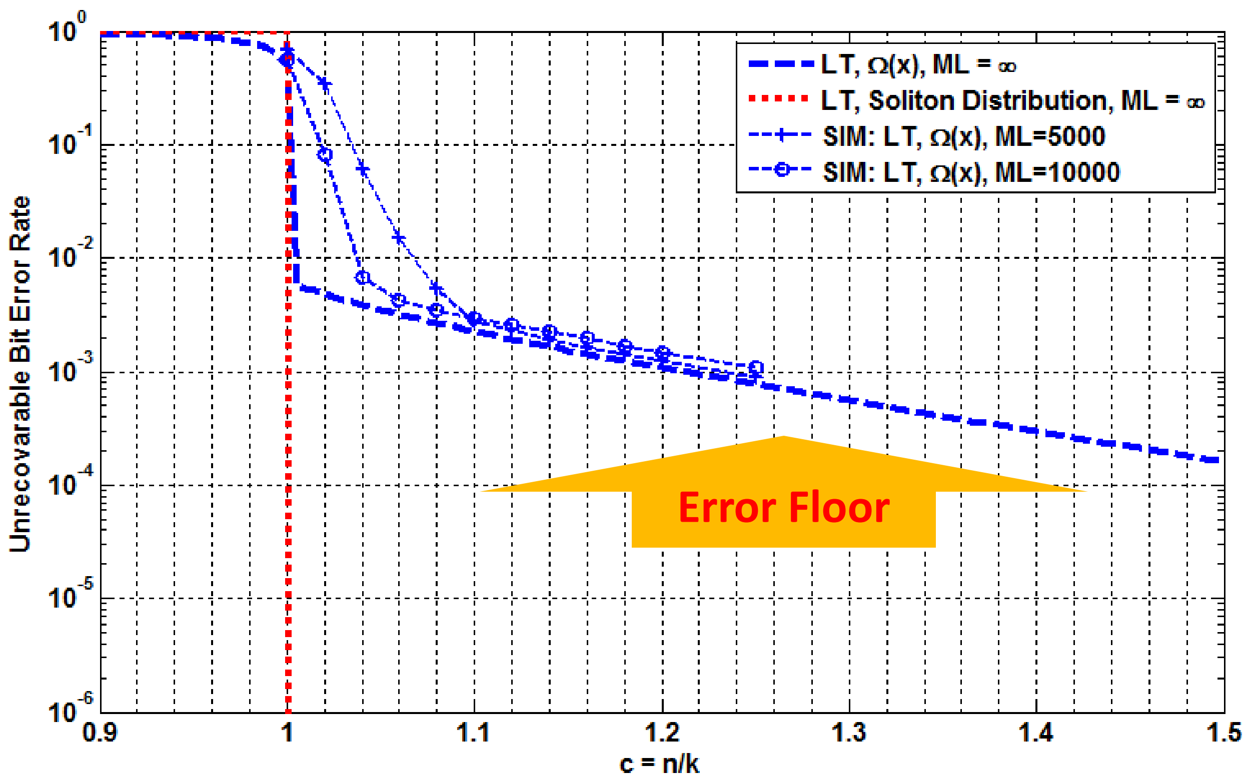
Efficiency & Complexity

- It is shown that using RSD, the probability that the decoding process will succeed after decoding $k + O(\sqrt{k}\ \ln^2(k/\gamma))$ is $1 - \gamma$. $\epsilon = O(\sqrt{k}\ \ln^2(k/\gamma))/k$
- The complexity of decoding is related to average number of XOR operations i.e., average number of edges in the graph $O\left(k \ln\left(\frac{k}{\gamma}\right)\right)$.
- **OBSERVATIONS:**
  - *Rateless construction.*
  - *Encoding/Decoding is <u>not linear</u> in k.*
  - *Asymptotically optimal (MDS) i.e., requires large n for vanishing ϵ.*
  - *We do not need to access all data symbols to resurrect a particular lost code symbol.*

How to achieve linear complexity?

- How about we have a distribution that the maximum degree is fixed, i.e., does not scale with increasing k.
- Let us assume we have the following degree distribution:

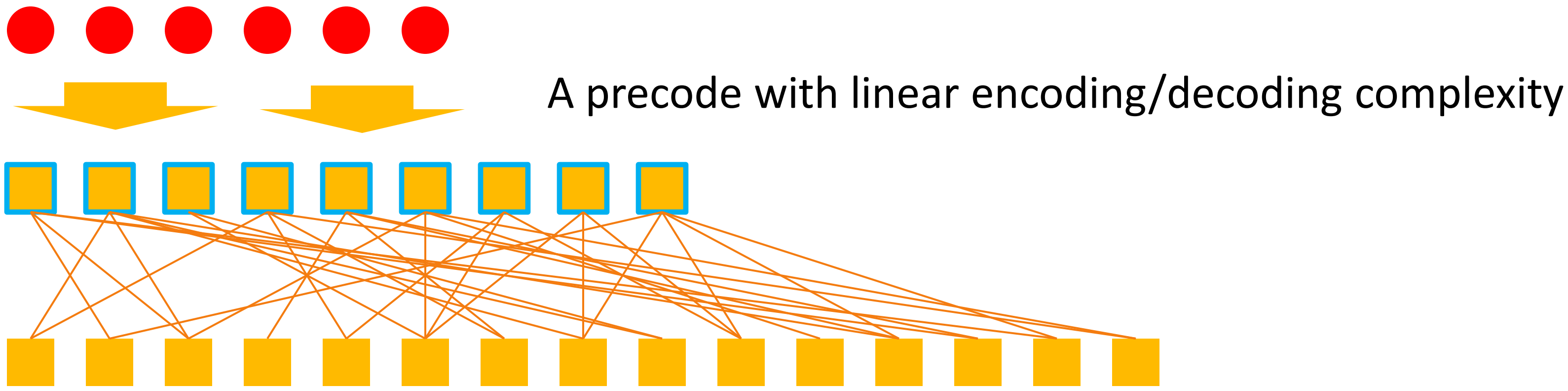
$$\Omega(x) = 0.007969x + 0.49357x^2 + 0.1662x^3 + 0.072646\,x^4 + 0.082558x^5 + 0.056058x^8$$
$$+0.037229x^9 + 0.05559x^{19} + 0.025023x^{65} + 0.003135x^{66}$$



[after erasures]

How to achieve linear complexity and no error floor?

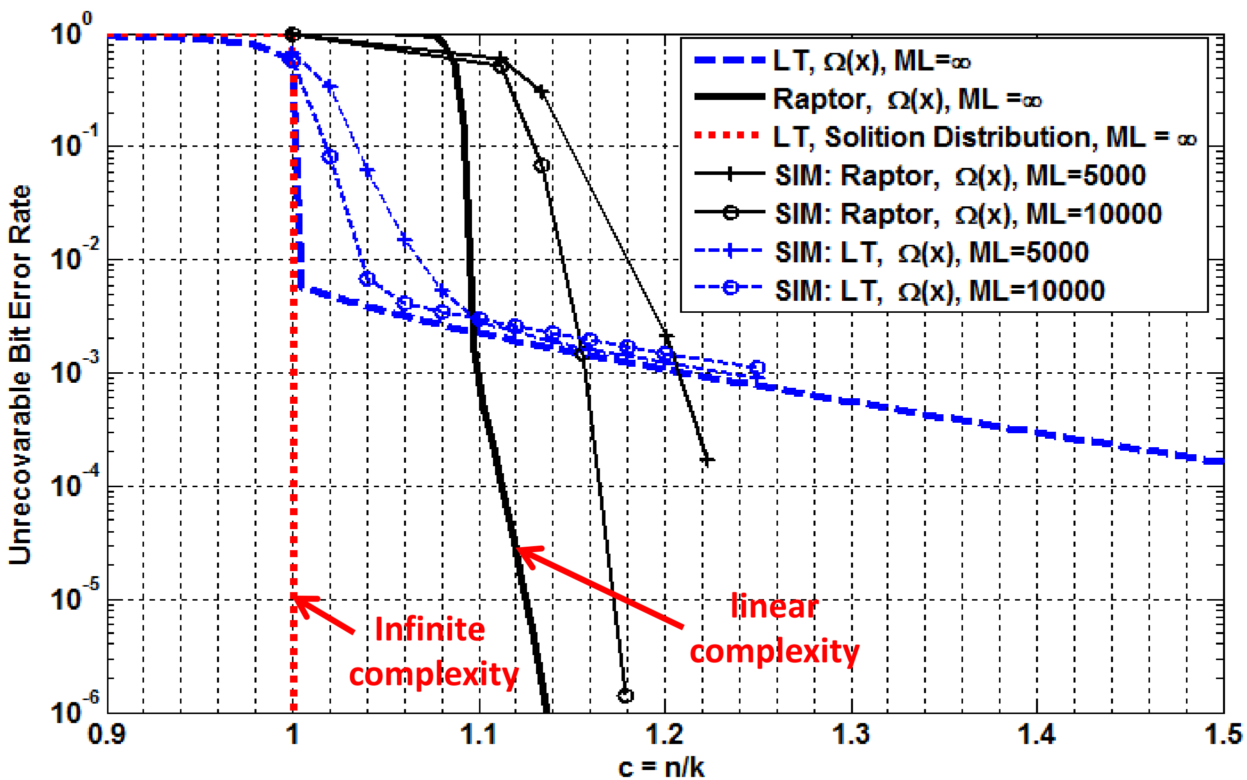
- The way to go is concatenation: Raptor Codes.



- Decoder for LT code decodes up to some fraction of the input symbols and then the rest of the erasures are corrected by the precode.
- The overall complexity is linear with k
- However, the overhead is increased due to the additional coding stage.

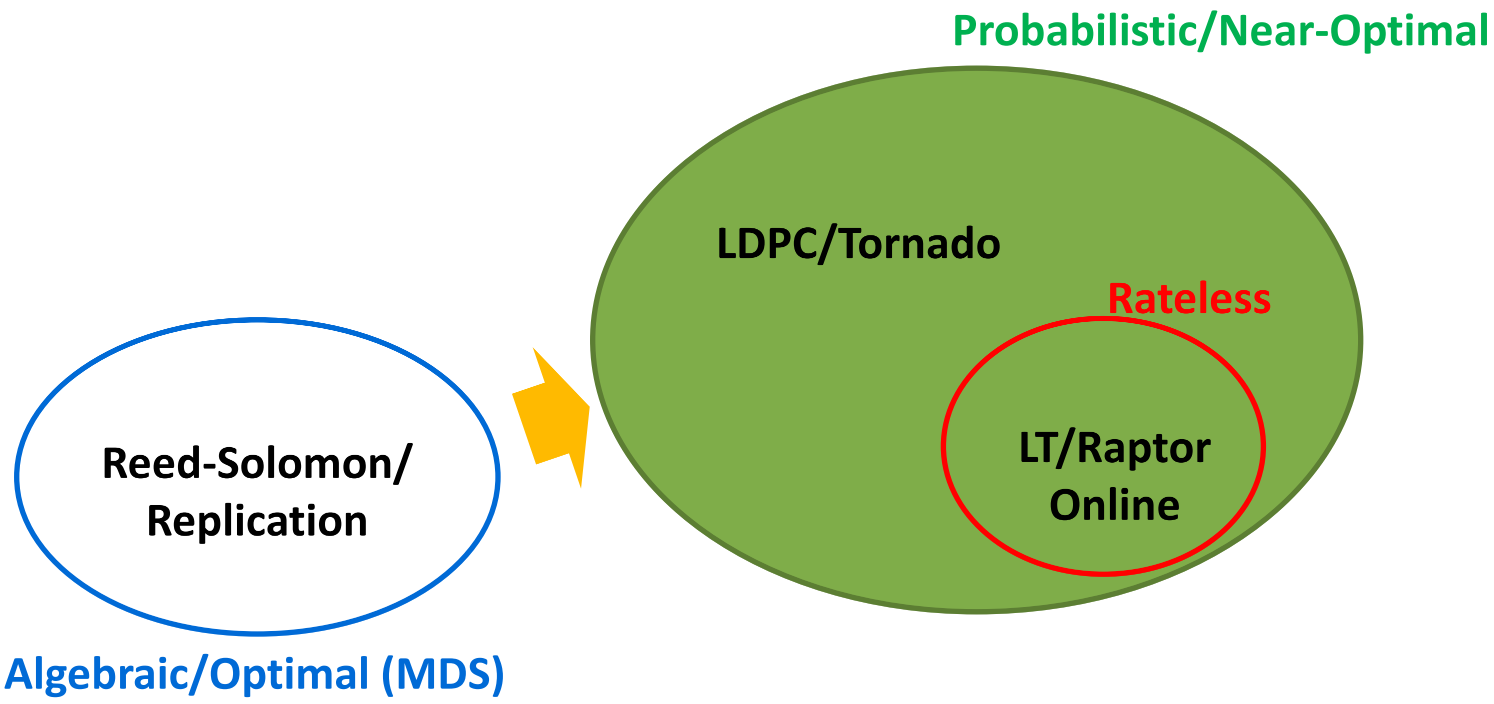
How to achieve linear complexity and no error floor?

- Raptor codes do not only achieve linear complexity, but also achieve low over head, near-optimal performance.
- They become to be part of 3rd Generation Partnership Project for use in mobile cellular wireless broadcast and also used by DVB-H standards for IP datacast to handheld devices



[after erasures]

Evolution of Erasure Codes



Quantum

ERASURE CODES FOR LARGE SCALE STORAGE

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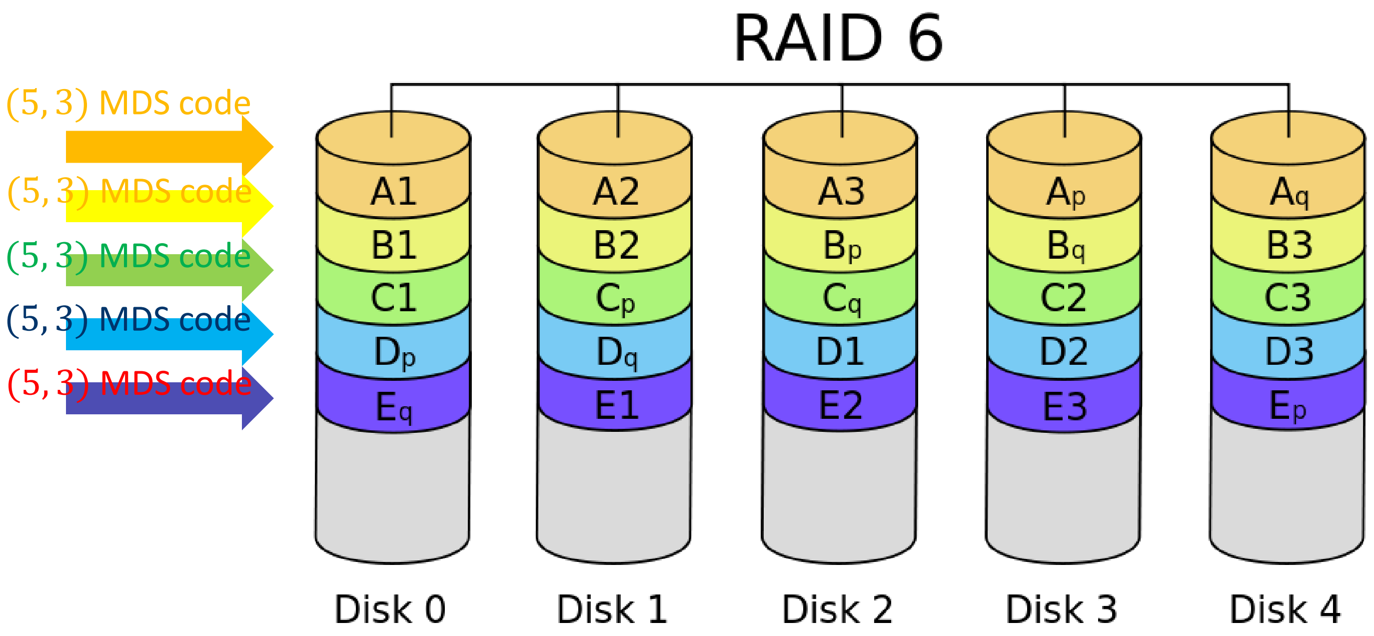
Evolution of erasure codes for data storage

- Classical MDS erasure codes are suboptimal for distributed storage networks because of the "*repair problem*"



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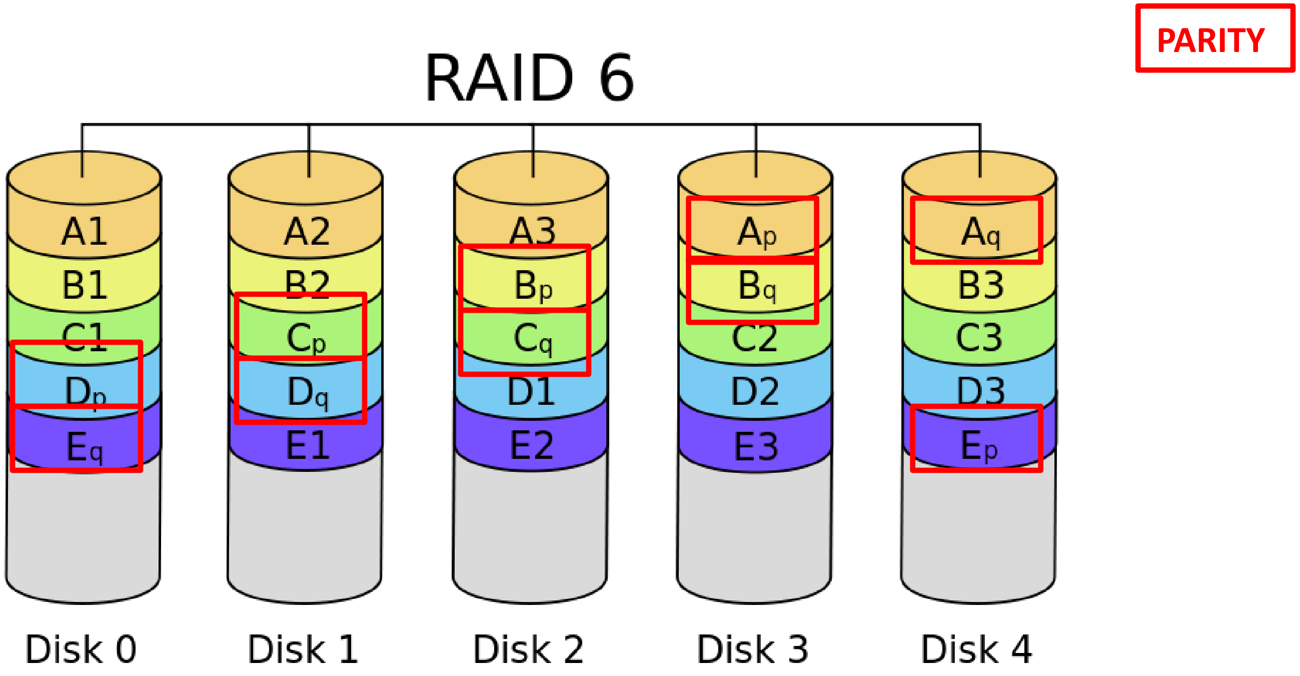

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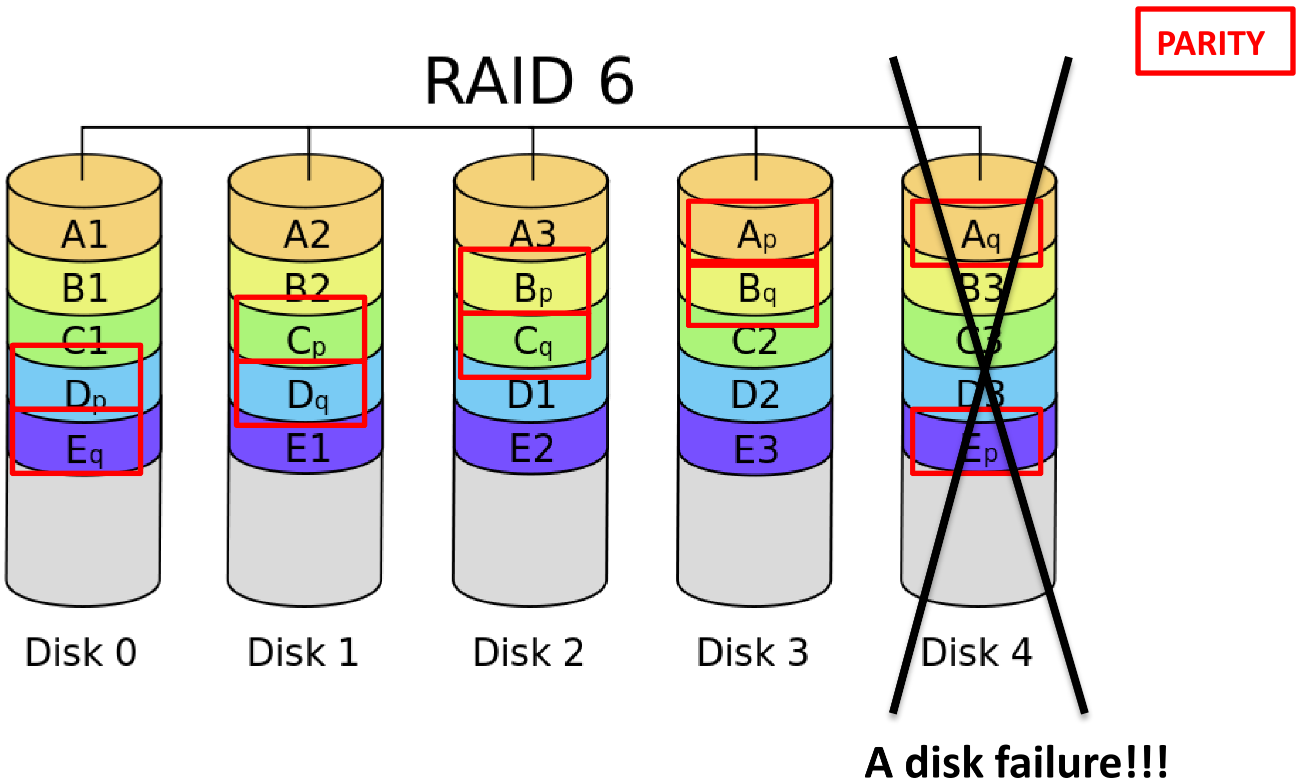

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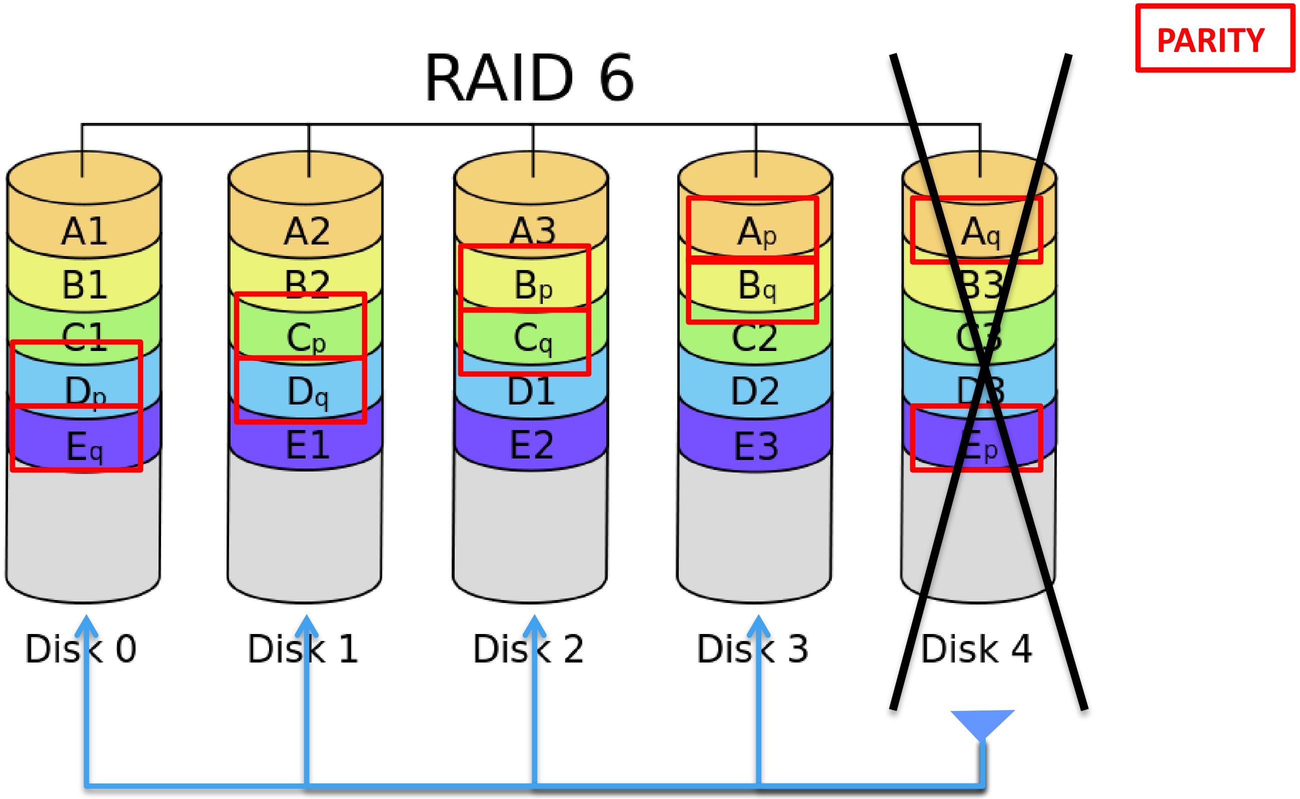


To repair the disk, we need to access all data in other disks: 4X more network repair overhead

Evolution of erasure codes for data storage

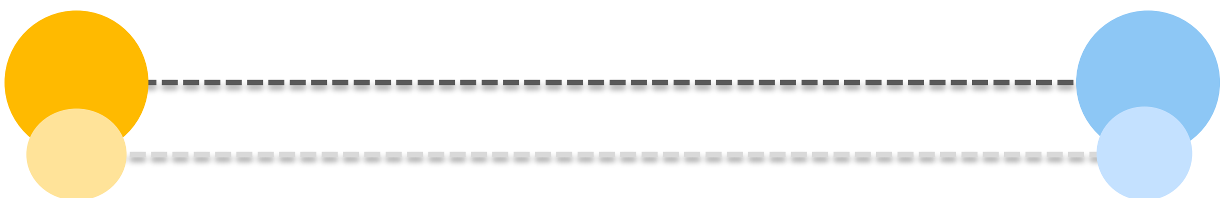
- Classical MDS erasure codes are suboptimal for distributed storage networks because of the *“repair problem”*.
- We need efficiently repairable erasure codes.

Least Efficient

Replication
(Repetition coding)

Most Efficient

Algebraic MDS codes



Fastest access/repair

Slowest access/repair

Evolution of erasure codes for data storage

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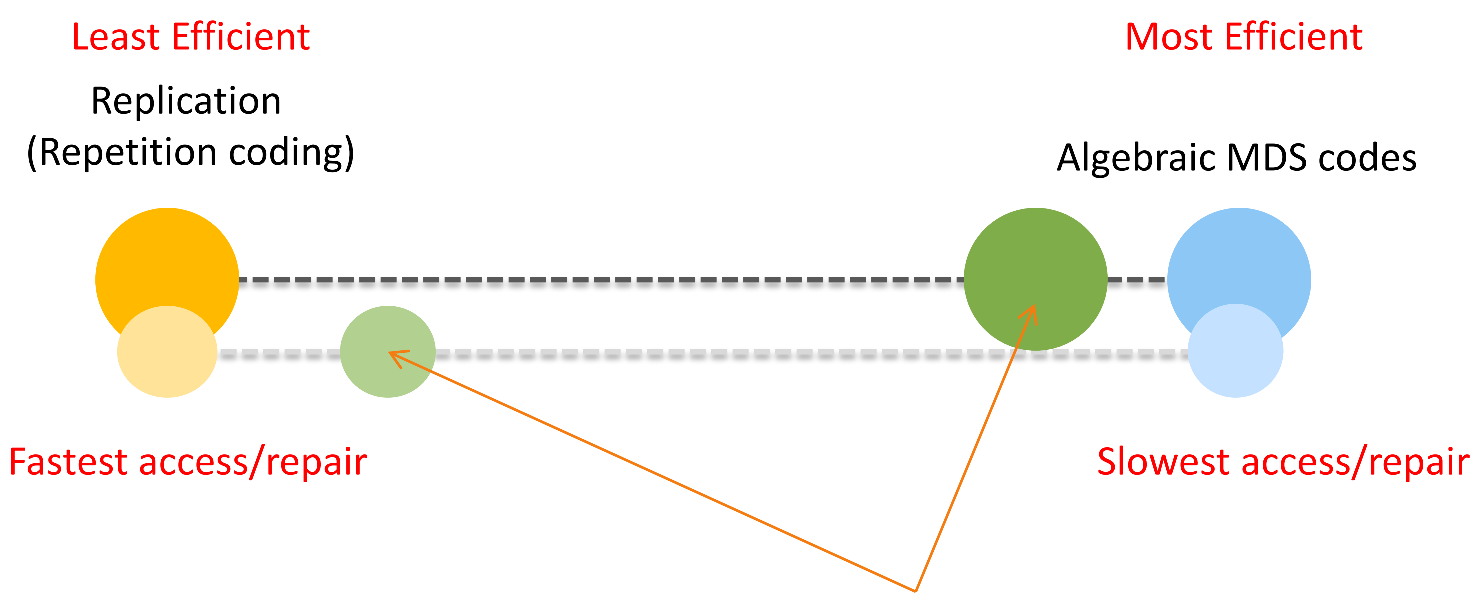
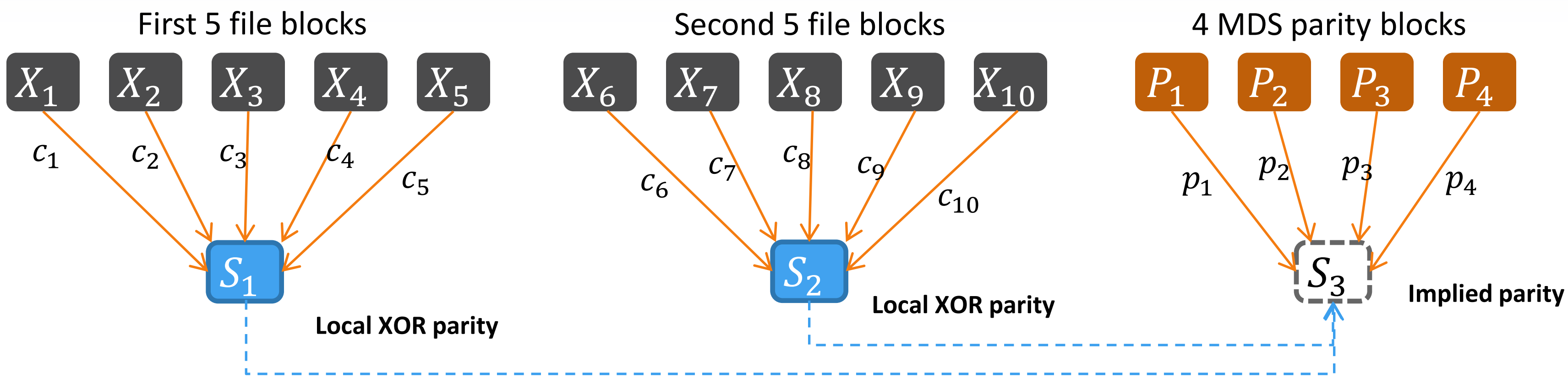


**With the evolution of erasure codes,
A small sacrifice in performance leads to efficiently
repairable good codes.**

A Case study – Facebook File system

[Locally repairable codes]

- Let us use a standard (14,10) MDS code.



- Calculate local parities:
  - $S_1 = c_1X_1 + c_2X_2 + c_3X_3 + c_4X_4 + c_5X_5$
  - $S_2 = c_6X_6 + c_7X_7 + c_8X_8 + c_9X_9 + c_{10}X_{10}$
- Also choose coefficients such that
  - $S_1 + S_2 = S_3$ [Implied parity – Not stored!]
- DISADVANTAGE : **Extra Storage Requirement!!!**

Overhead (RS) = 14/10 = 1.4

Overhead (LRC)= 14/10 = 1.6

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[Locally Repairable Codes - LRC]

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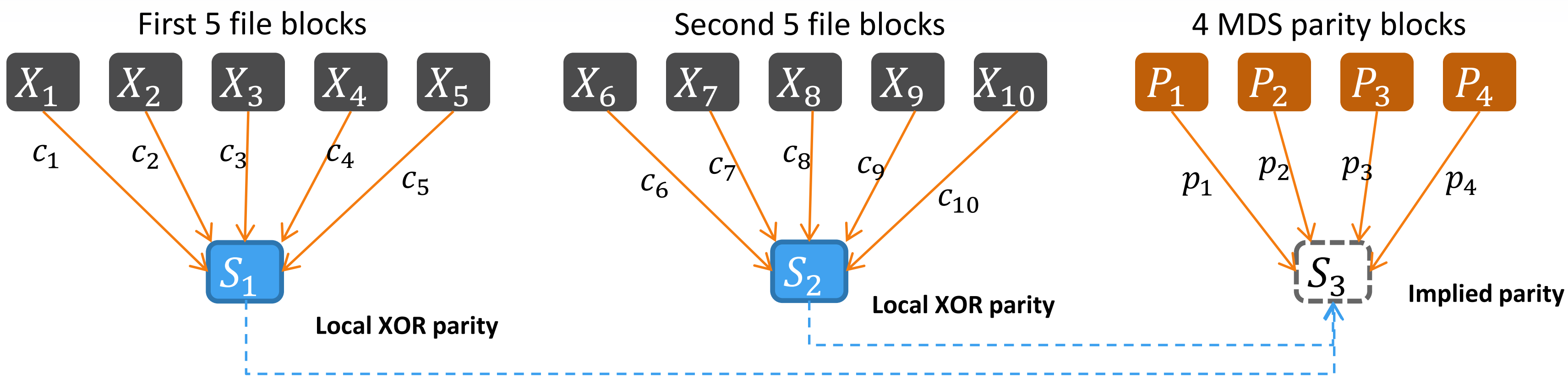

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- Also choose coefficients such that
  - $S_1 + S_2 = S_3$ [Implied parity – Not stored!]
- DISADVANTAGE : **Extra Storage Requirement!!!**

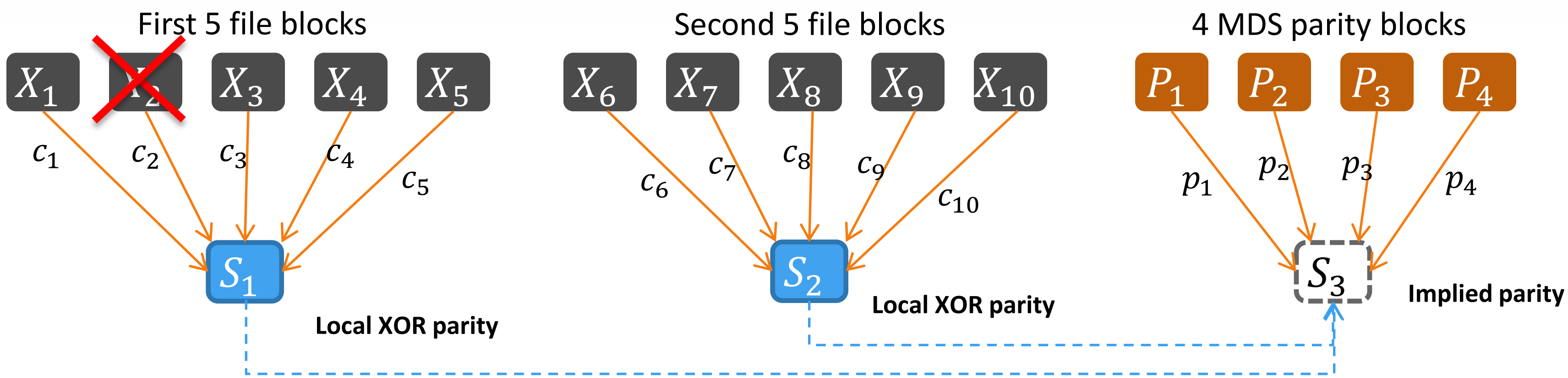
Overhead (RS) = 14/10 = 1.4

Overhead (LRC)= 16/10 = 1.6

A Case study – Facebook File system

[Locally Repairable Codes - LRC]

- **Suppose a failure occurs!**

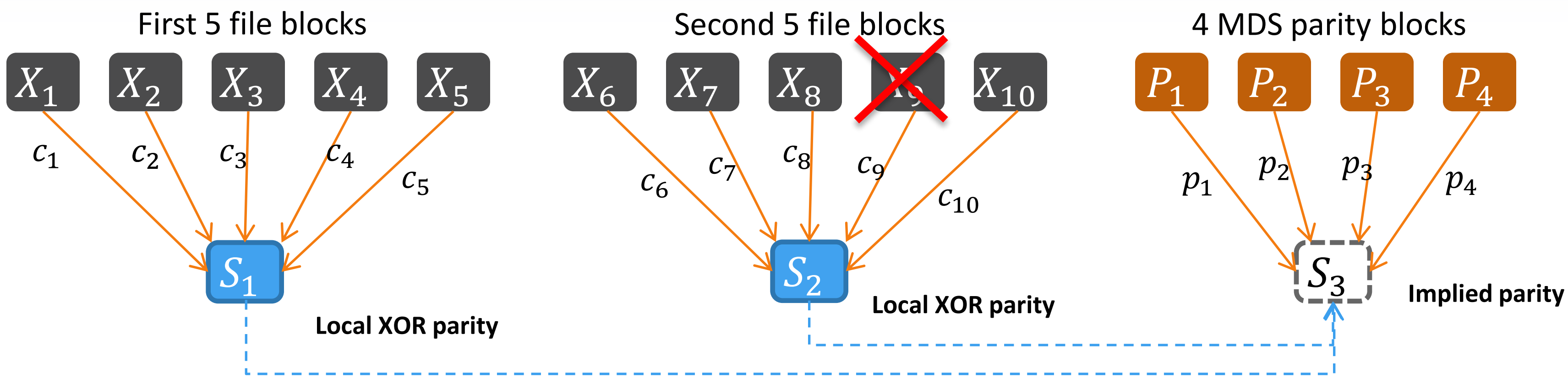


- Recovery equation:
  - $X_2 = {c_2}^{-1}(S_1 - c_1X_1 - c_3X_3 - c_4X_4 - c_5X_5)$
- Number of blocks that need to be accessed and read: 5!

A Case study – Facebook File system

[Locally Repairable Codes - LRC]

- **Suppose a failure occurs!**

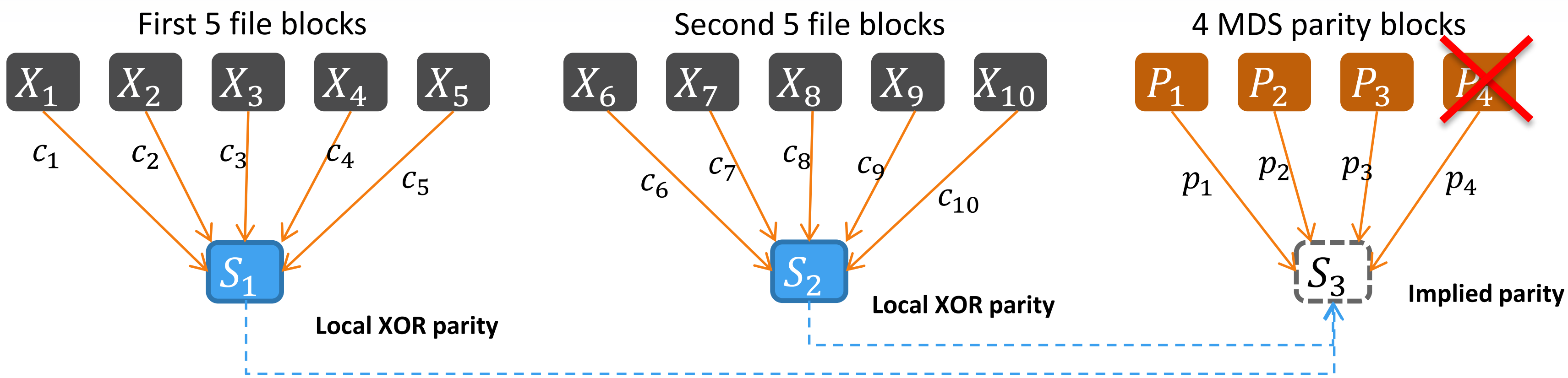


- Recovery equation:
  - $X_9 = c_9^{-1}(S_2 - c_6X_6 - c_7X_7 - c_8X_8 - c_{10}X_{10})$
- Number of blocks that need to be accessed and read: 5!

A Case study – Facebook File system

[Locally Repairable Codes - LRC]

- Suppose a failure occurs!

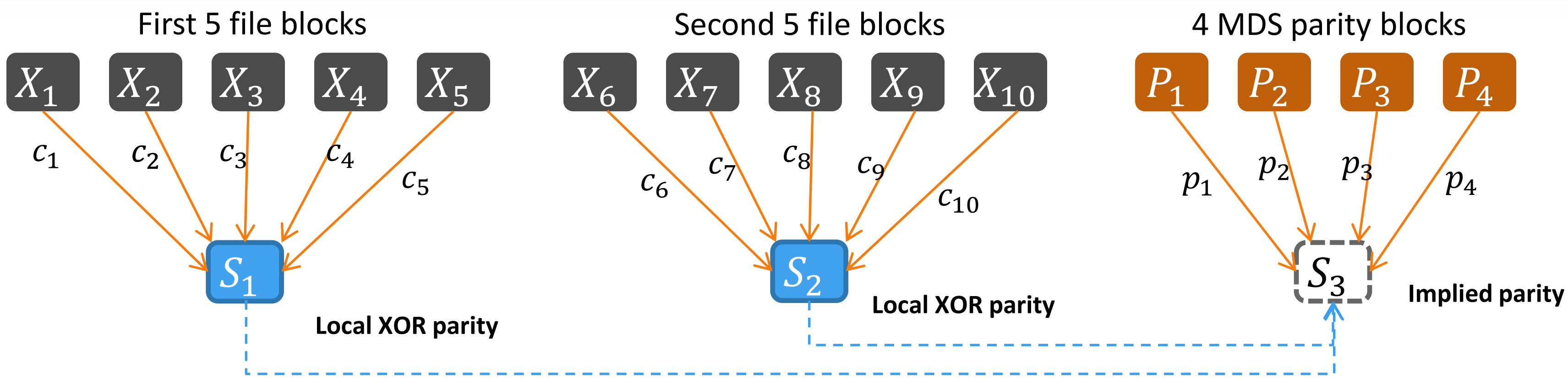


- Recovery equation:
  - $P_4 = {p_4}^{-1}(-S_1 - S_2 - p_1 P_1 - p_2 P_2 - p_3 P_3)$
- Number of blocks that need to be accessed and read: 5!

A Case study – Facebook File system

[Locally Repairable Codes - LRC]

- **Suppose a failure occurs!**



- Single failure : MIN:5 , MAX:5
- Double failures: MIN:9, MAX: 12
- Tripple failures: MIN: 12, MAX:12

LRC (16, 10)

- Single failure : MIN:12, MAX:12
- Double failures: MIN:12, MAX: 12
- Tripple failures: MIN: 12, MAX:12

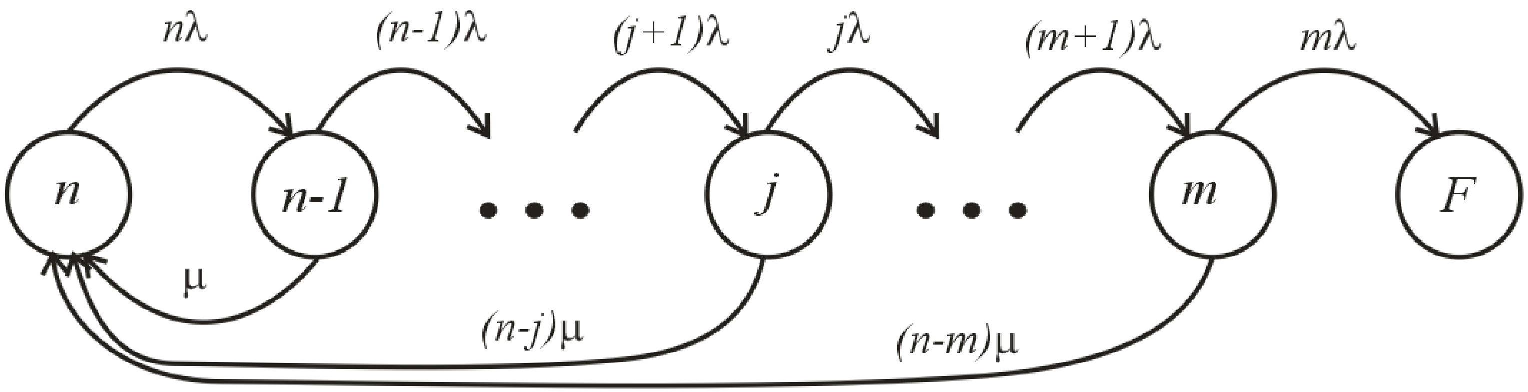
RS (14, 10)

Block locality

- **Definition**: (**Block locality**) An (n, k) code has a block locality l, when each block/unit is a function of at most l other blocks.
- Example: An (n, k) RS code has a block locality of k.
- We desire erasure codes with block locality $l \ll k$
- **Theorem 3**: (**locally repairable codes (LRC)**) There exists (n, k) locally repairable codes with block locality $\ln(k)$ that can correct $n - (1 + \epsilon)k$ erasures where $\epsilon = \frac{1}{\ln(k)} - \frac{1}{k}$.
  - **Example:** LT codes has an average symbol degree of $\ln(k)$ and therefore has an average block locality of $\ln(k)$ while achieving an optimal performance asymptotically.

How all these measures reflect to system performance?

- A storage system's reliability is usually measured in terms of mean time to failure (MTTDL) values.
- Assume we have $\boldsymbol{n}$ disks, $\boldsymbol{m}$ of which are used for data storage and $\boldsymbol{c} = \boldsymbol{n} - \boldsymbol{m}$ are used parity (failure protection).
- Conventionally, a Markov model is used (with some correction factors) to predict the MTTDL values.



- Each state represents the number of operational disks in the array. Transitions happen with each component having constant failure and repair rates λ and μ, respectively.

Reliability and the Markov Model

- This model assumes an $(n,\ m)$ MDS code that can correct up to $c\ =\ n-m$ erasures.
  - ASSUMPTIONS:
    1) Disk failures are independent
    2) Each disk failure and repair happens based on an exponential distribution (Poisson random process).
- **MTTDL is the expected time to enter state F.**

$$P_i(t)\text{: } probability\ of\ being\ in\ state\ i\ at\ time\ t$$

Reliability function

$$R(t) = \sum_{j=m}^{m+c} P_j(t)$$

MTTDL

$$MTTDL = \int_0^{\infty} R(t)dt$$

- Slight changes (a single disk repair at a time) can be made to the model, however these changes only slightly effect the MTTDL value.

Reliability and the Markov Model

- Due to assumption 1 and 2,
  - $MTTF = 1/\lambda$ and $MTTR = 1/\mu$
- Let $\omega = \dfrac{\mu}{\lambda} = \dfrac{MTTF}{MTTR}$.
- We have,

$$MTTDL = \frac{\omega^c}{\lambda\, m \binom{m+c}{c}}$$ **[1]**

- For $c = 1$ (RAID 5)

$$MTTDL = \frac{\mu}{\lambda^2\, m\,(m+1)} = \frac{MTTF^2}{m\,(m+1)MTTR}$$

- For $c = 2$ (RAID 6)

$$MTTDL = \frac{\mu^2}{\lambda^3\, \frac{1}{2} m\,(m+1)(m+2)} = \frac{MTTF^3}{\frac{1}{2} m\,(m+1)(m+2)MTTR^2}$$

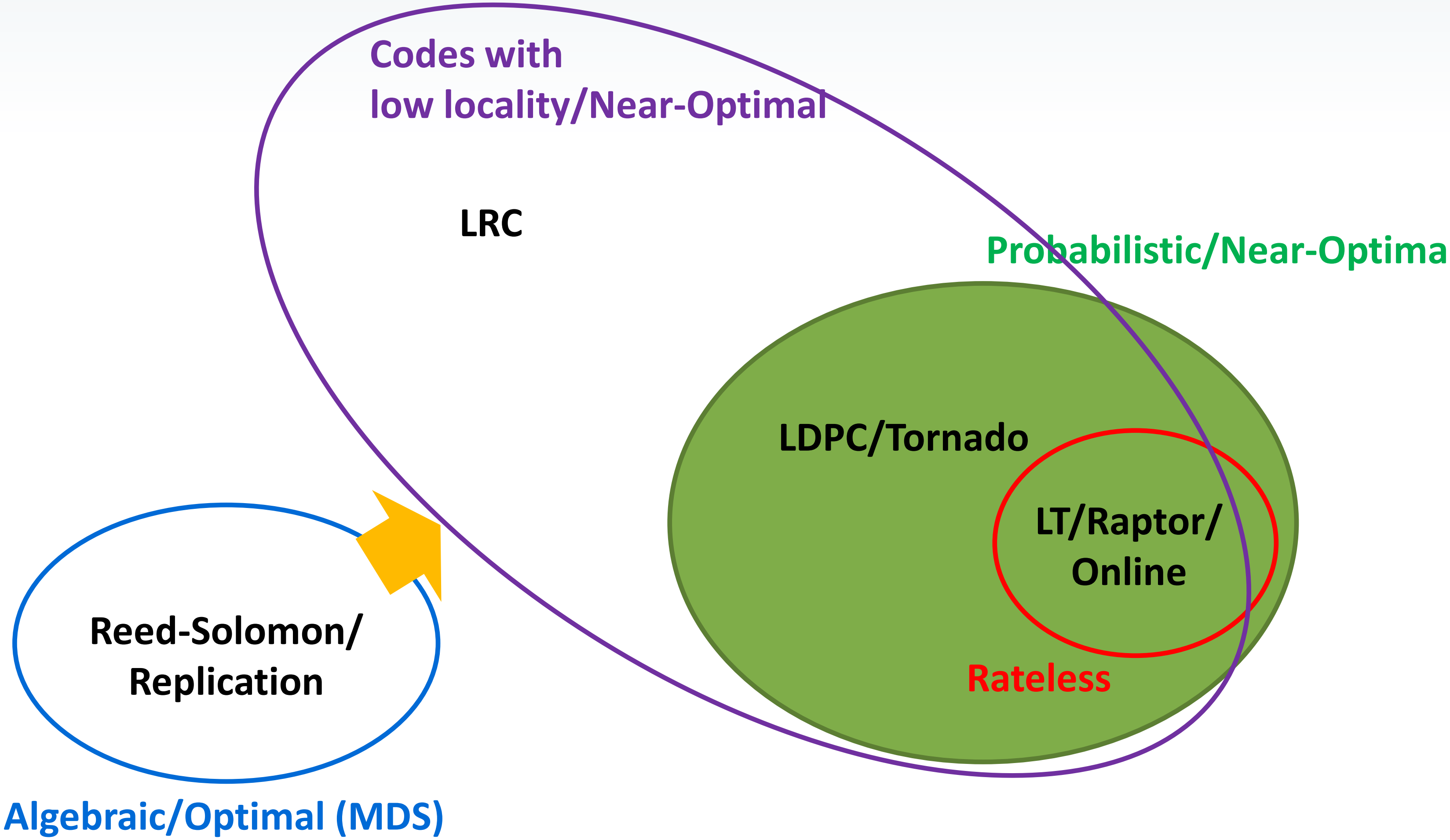
[2] W. Burkhard, and J. Menon, "Disk array storage system reliability". *Proceedings of the International Symposium on Fault-tolerant Computing*, pgs.432-441, 1993..

Numerical Results [2]

Erasure Code	Storage overhead	Repair traffic	MTTDL (days)
Replication (3, 1)	3x	1x	2.3e+10
RS (14, 10) – optimal	1.4x	10x	3.3e+10
LRC (16, 10)	1.6x	5x	1.2e+15

[3] M. Sathiamoorthy, M. Asteris, D.S. Papailiopoulos, A.G. Dimakis, R. Vadali, S. Chen, and D. Borthakur. Xoring elephants: Novel erasure codes for big data. In Proceedings of the VLDB Endowment, 2013

Evolution of Erasure Codes



OPTIMIZATION OF LT CODES FOR IMAGE TRANSFER

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Source quality assessment: Image compression

-Basics

- Given two images ***I*** and ***I'*** (original and the noisy version), the distortion will be measured by Mean Square Error (MSE):

$$MSE = \frac{1}{L_x \times L_y} \sum_{y=1}^{L_x} \sum_{x=1}^{L_y} [I(x,y) - I'(x,y)]^2$$

where L_x and L_y are dimensions of the image.

- Peak Signal to Noise Ratio (PSNR in dB) is defined to be

$$PSNR = 10 \times \log_{10} \left(\frac{I_{max}^2}{MSE} \right)$$

where I_{max} is the maximum possible intensity value of the image.

- For monochromatic gray scale image: $I_{max} = 255$
- Lower MSE (larger PSNR) means better image quality.
- "Source rate" means the average number of bits spent per pixel (bpp). For a given PSNR value, the lower the source rate is, the better the compression will be.

Progressive Source Compression

-Introduction

Progressive bit stream

0.01bpp, PSNR=22.55dB

- 4% gives you only a low quality representation of the source.

Progressive Source Compression

-Introduction

Progressive bit stream

0.01bpp, PSNR=22.55dB

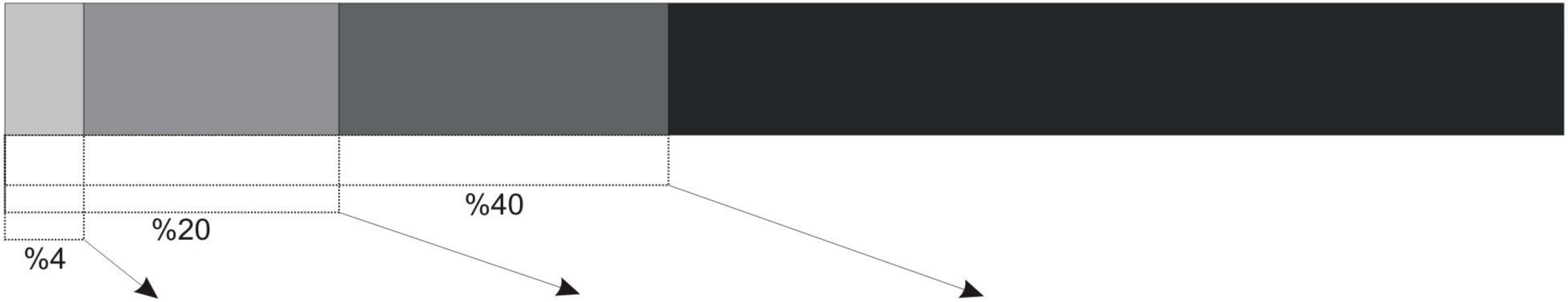
0.05bpp, PSNR=27.17dB

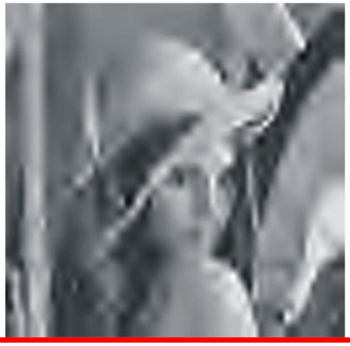
- 20% gives better image quality compared to 4% case.

Progressive Source Compression

-Introduction

Progressive bit stream





0.01bpp, PSNR=22.55dB

0.05bpp, PSNR=27.17dB

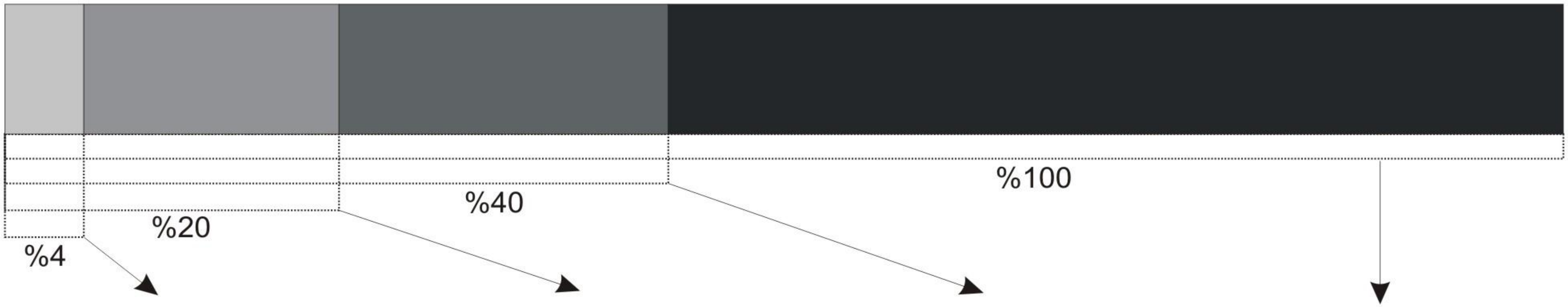
0.1bpp, PSNR=29.81dB

- Using only 40% of the total bit stream, a good quality image is obtained.

Progressive Source Compression

-Introduction

Progressive bit stream



0.01bpp, PSNR=22.55dB

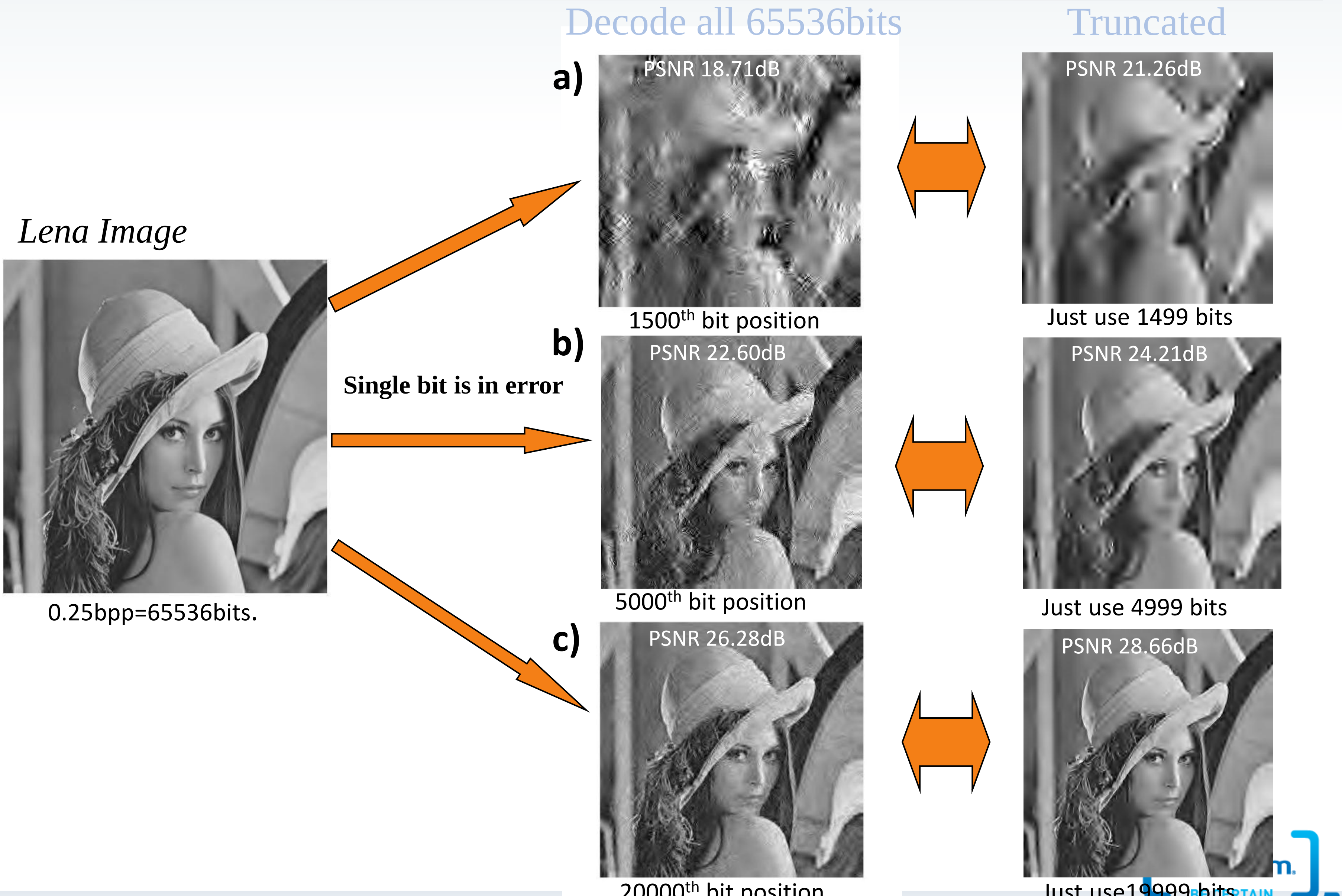
0.05bpp, PSNR=27.17dB

0.1bpp, PSNR=29.81dB

0.25bpp, PSNR=33.68dB

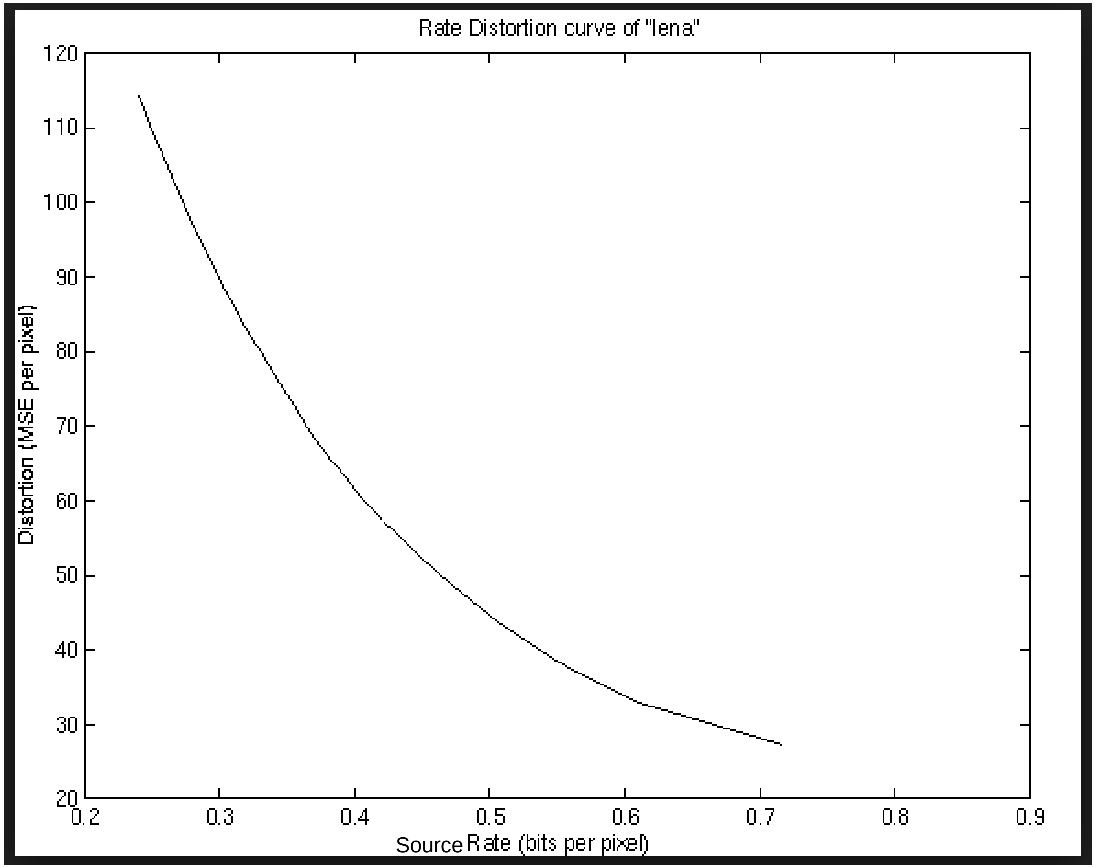
- At 100%, the image quality is improved further but no major difference from 40%.
- Examples: SPIHT, EZW, JPEG2000 etc.
- Disadvantage: Very sensitive to bit errors.
- Unequal Error Protection (UEP) is achieved by channel coding.

Error Propagation



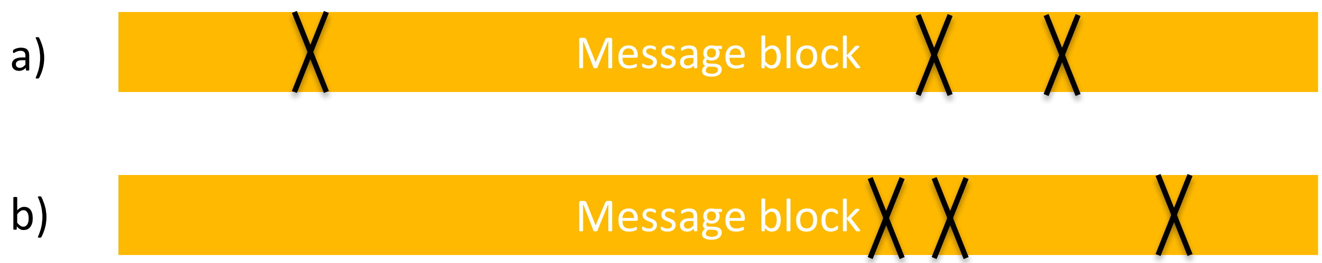
Rate-Distortion Curve

- In a lossy data compression, R-D curve pictures the relationship between the source rate and the distortion for a given source and encoder/decoder pair.



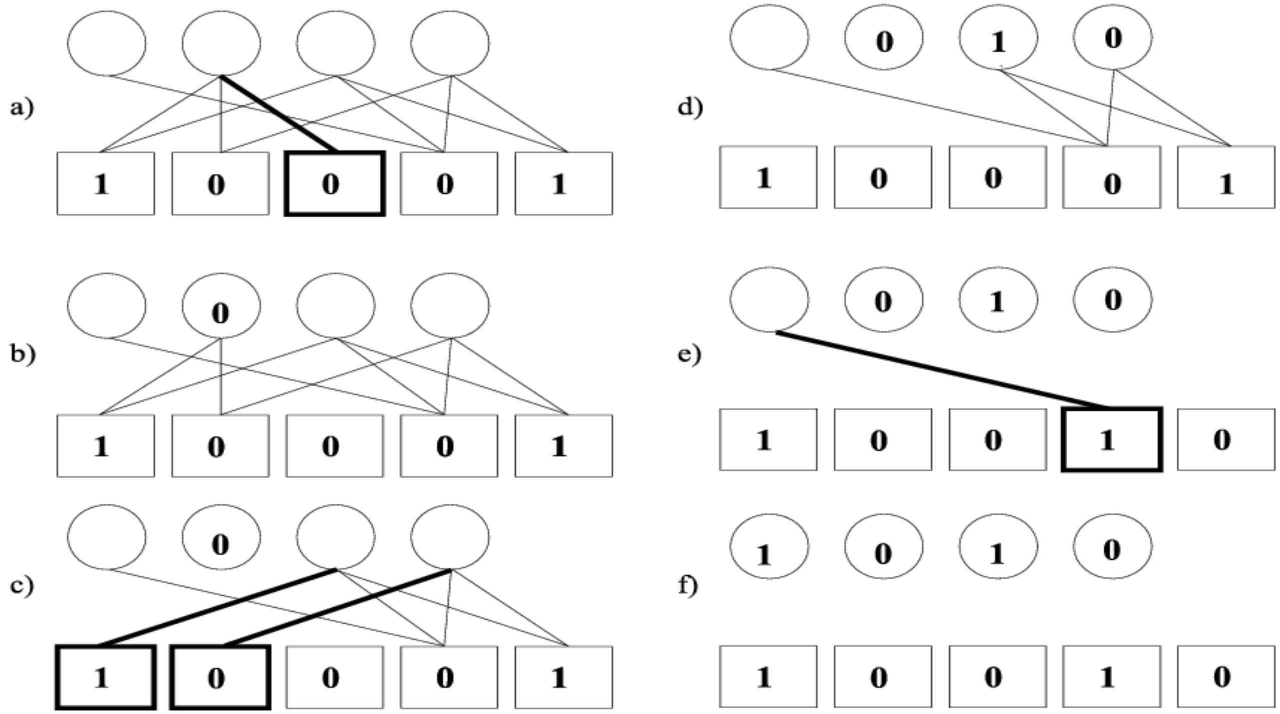
Design objective of the erasure code

- Decoding the whole message block? If $c < 1$, this is not even possible with optimal codes!
- Decoding a fraction of the message block? What fraction?



- Both have the same number of unrecoverable errors. However **b)** will provide better image quality!
- Need unequal protection/unequal recovery time.

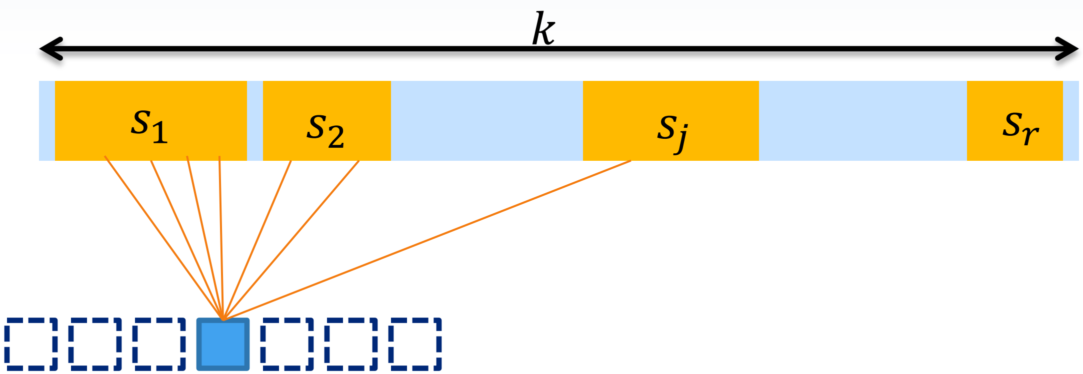
Belief Propagation



- Let us observe the following:
  - *Decoding stage 1:* A degree-1 check node decodes an information symbol.
  - *Decoding stage 2:* Some of the degree-2 check nodes decode two information symbols.
  - *Decoding stage 3:* A degree-3 check node decodes an information symbol.
- **Conclusion:** low degree coded symbols decode information symbols earlier (early iterations) in the decoding algorithm.
- This can be used for prioritized decoding.

Idea

- First step: divide the message block into multiple subblocks (r subblocks).



$d = 7$

- Second step: For each symbol generated: Choose a degree according to a suitable degree distribution.
- Let $p_{j,i}$ be the conditional probability of choosing any information symbol in s_j given the degree of the coded symbol is i.

$$\mathbf{P}_{r\times k} = \begin{bmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,k} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{r-1,1} & p_{r-1,2} & \cdots & p_{r-1,k} \\ p_{r,1} & p_{r,2} & \cdots & p_{r,k} \end{bmatrix}$$

[4] S. S. Arslan, P. Cosman, and L. Milstein, “Generalized unequal error protection LT codes for progressive data transmission,” *IEEE Trans. Image Processing*, vol. 21, no. 8, pp. 3586–3597, Aug. 2012.

- Second step: Choose edges according to $\mathbf{P}_{r\ x\ k}$

Idea

- Number of unknowns: $(r-1)k$ i.e., it scales with k.
- To reduce the number of unknowns, we introduce an exponential dependence:

Motivation: *exponential-like RD characteristics*

choose $p_{j,i}$ to be an exponential function of the degree number i for $j = 1, 2, \ldots, r-1$ as follows:

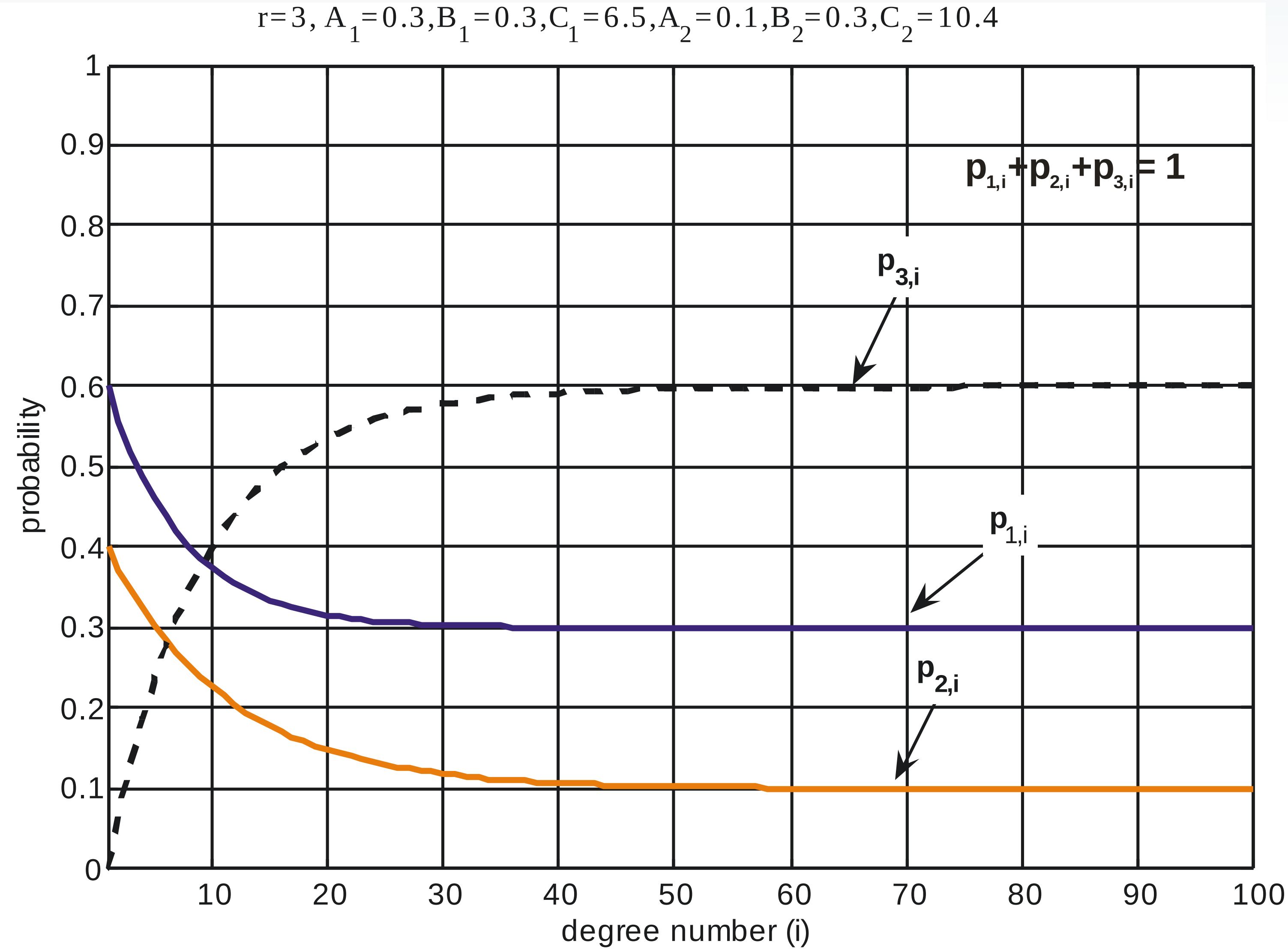
Definition *Exponential* SD

- $$p_{j,i} = A_j + B_j \times \exp\left\{-\frac{i-1}{C_j}\right\} \text{ for } i = 1, 2, \ldots, k$$

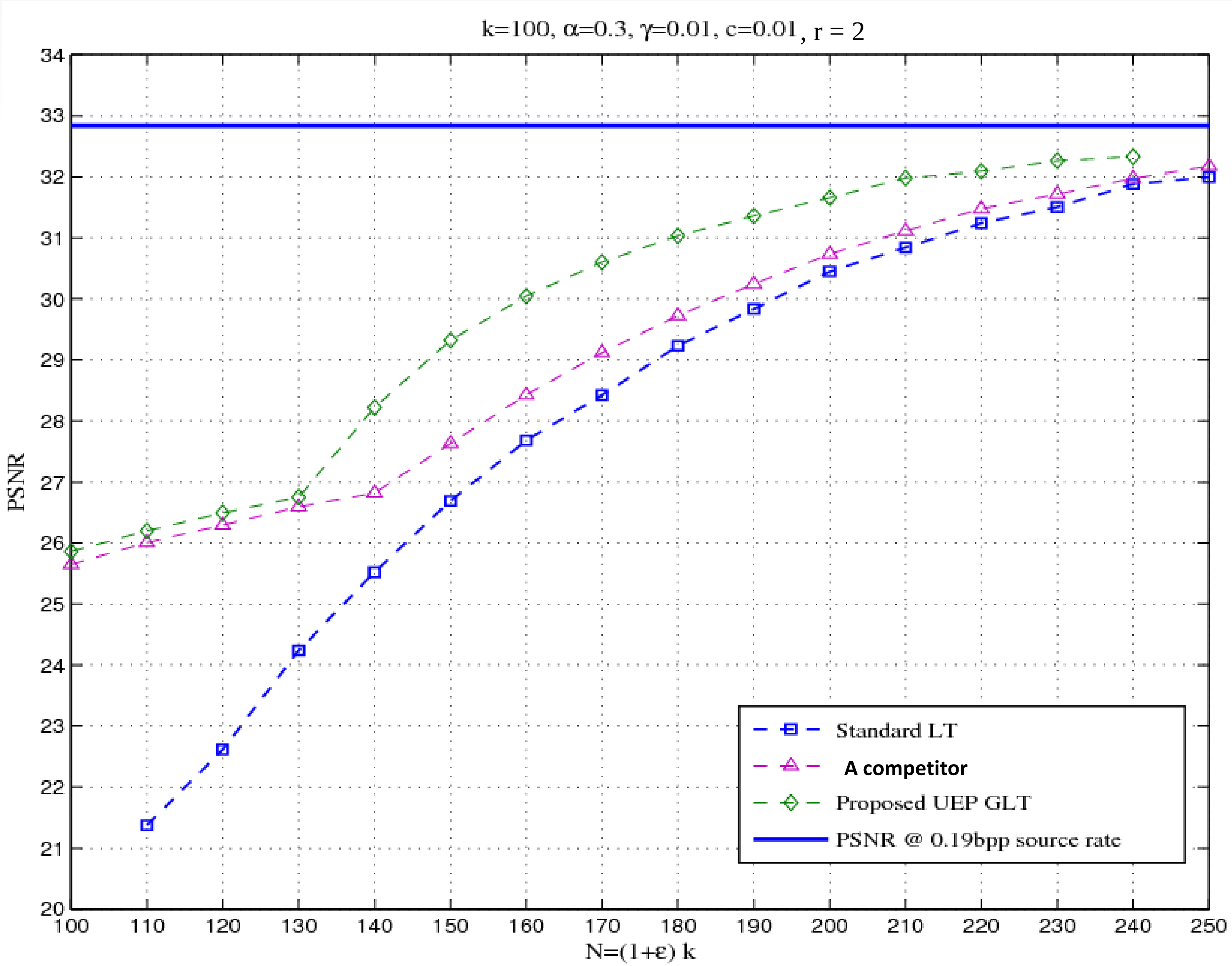
where $\{A_j \geq 0, B_j \geq 0, C_j \geq 0\}_{j=1}^{r-1}$ are design parameters satisfying $\sum_{j=1}^{r} p_{j,i} = 1$ for all i.

- Number of parameters are reduced to $3(r-1)$.

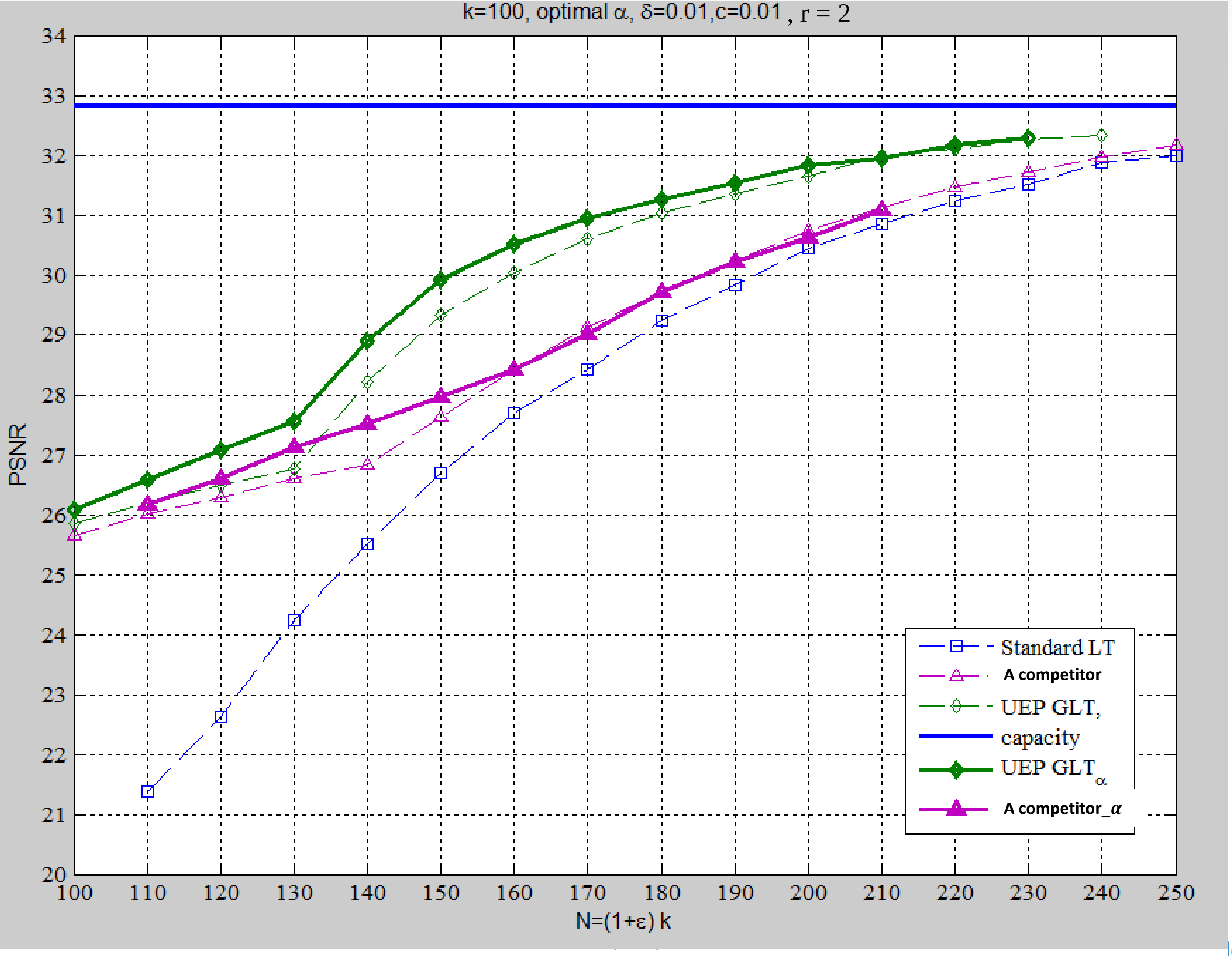
Idea


r=3, A_1=0.3,B_1=0.3,C_1=6.5,A_2=0.1,B_2=0.3,C_2=10.4
p_1,i+p_2,i+p_3,i= 1
p_3,i
p_1,i
p_2,i
probability
degree number (i)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
10
20
30
40
50
60
70
80
90
100

Simulation result



Simulation result



Conclusions

- Depending on the application, the evolution of erasure codes have taken different directions.
- Different types of erasure codes are considered for different types of applications.
- For storage applications, main trend is to design codes with near-optimal performance in terms of efficiency with reduced bandwidth requirements while making sure that the error probabilities are under some target.
- For multimedia applications, main trend is to maximize the transfer multimedia quality or minimize the distortion.
- Optimization of the parameters of the erasure code can increase performance.

THANKS!

BE CERTAIN